Filters

* In this part we will discuss filters design and implementation

* **Ideal filter Response**

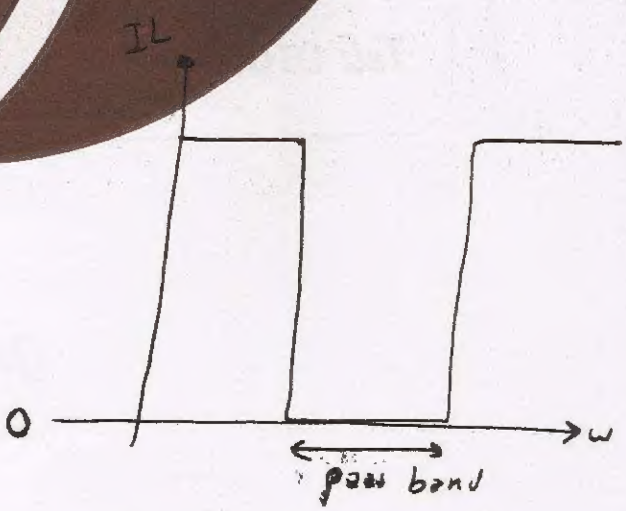
$P_{LR} \triangleq$ power loss ratio

$$= \frac{\text{Power available from source}}{\text{Power absorbed by the load}}$$

$$\boxed{P_{LR} = \frac{1}{1 - |\Gamma(\omega)|^2}}$$

IL ≜ The insertion loss

$$\boxed{IL = 10 \log_{10} P_{LR}}$$



Ideal response

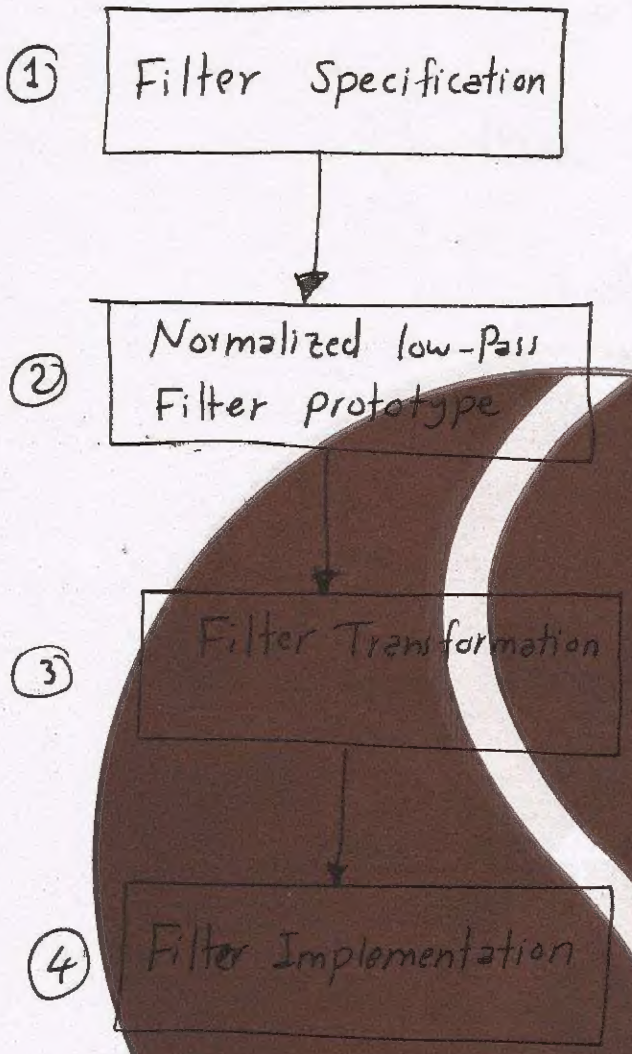
* Filter Design techniques:

1. Periodic structures
2. Image Parameter Method
3. Insertion loss Method

* Filter Implementation techniques:

1. Stubs with separating unit Elements
2. Stepped impedance low-Pass Filter
3. Coupled line filters
4. Coupled Resonator Filters.

* The flow chart of Realization Procedure :



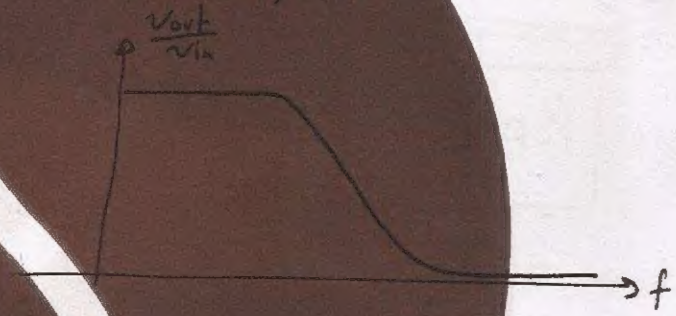
I) The Filter Specifications

$$\therefore \quad P_{LR} = \frac{1}{1 - |\Gamma(\omega)|^2} = f(\omega^2)$$

by equating the power loss ratio with different polynomial, we can obtain different filter response

I) Binomial "Butterworth" "Maximally flat"

$$\boxed{P_{LR} = 1 + K^2 \left(\frac{\omega}{\omega_c}\right)^{2N}}$$



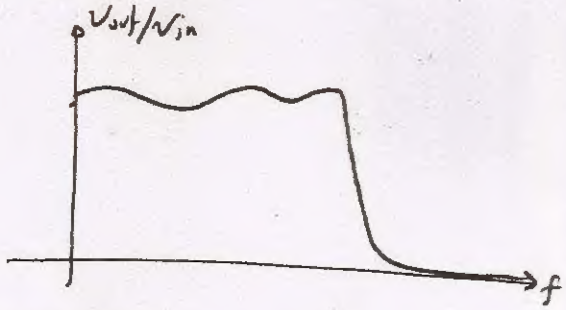
→ maximally flat PB

→ IL in SB "stop band" = 20 N dB/decade

→ slow transition

II) Chebychev type I

$$\boxed{P_{LR} = 1 + K^2 \, T_N^2\left(\frac{\omega}{\omega_c}\right)}$$



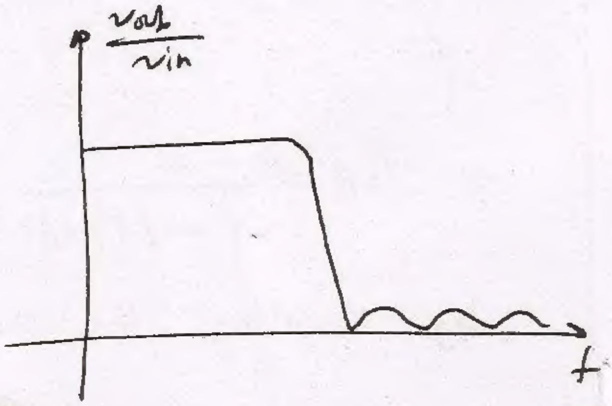
→ equal ripple in PB

→ IL in SB 20 N dB/decade "highest IL in SB"

→ sharp transition

(III) Inverse chebychev " chebychev type (2) "

$$\boxed{P_{LR} = 1 + \frac{K^2}{T_N^2\left(\frac{w_c}{w}\right)}}$$



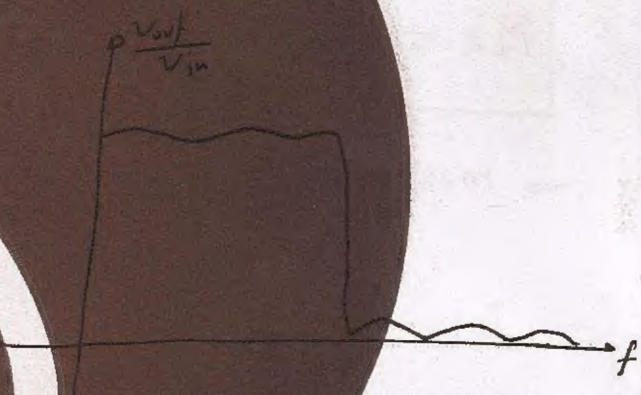
→ flat in PB

→ Equal ripple in SB

→ sharp Transition

(IV) Elliptic

$$\boxed{P_{LR} = 1 + K^2 R_N^2\left(\frac{w}{w_c}\right)}$$

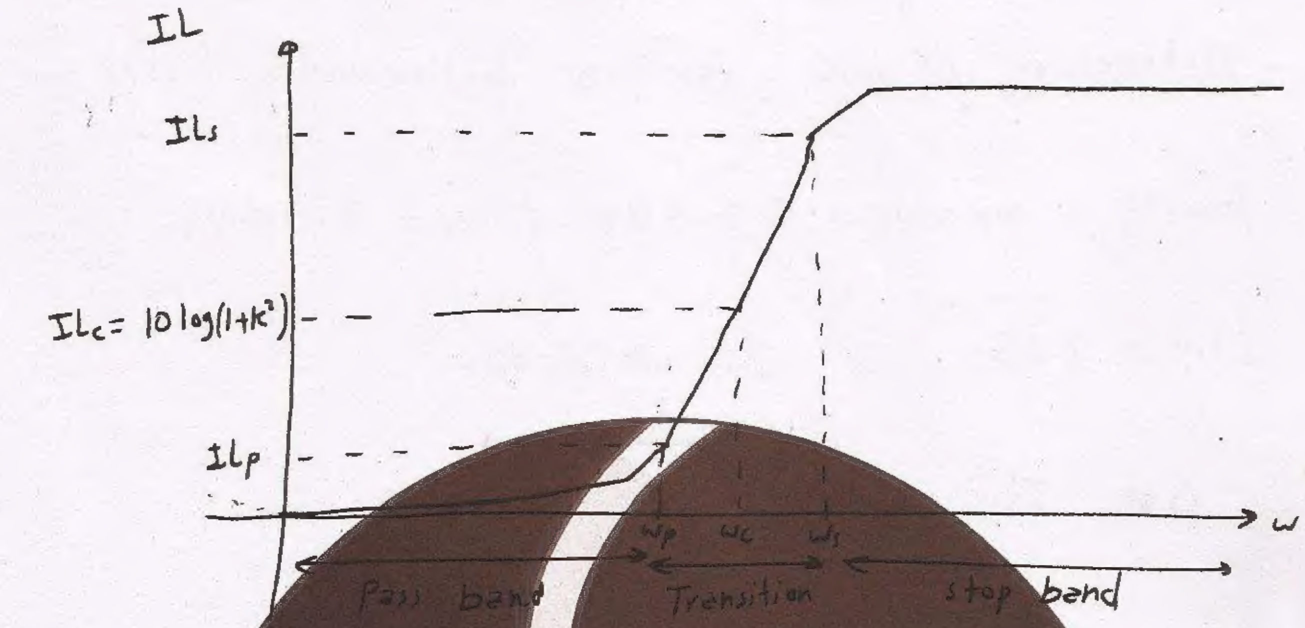


→ equal ripple in PB

→ equal ripple in SB

→ sharpest transition

* low pass filter prototype response



Given IL_p , IL_s $\Rightarrow$ we need to calculate

K , N [$N \equiv$ the filter order]

There are two Methods to obtain N, K

[1] Analitically

[2] Graphically

Example

- Determine N, K for a butterworth filter having $\omega_p = \omega_c = 5.5\ GHz$, $\omega_s = 8.9\ GHz$

$IL_p = 2\ dB$, $IL_s \geq 18\ dB$

- Get IL_s for your filter

(I) We will use the analytical solution

$$\boxed{P_{LR} = 1 + K^2 \left(\frac{\omega}{\omega_c}\right)^{2N}} \longrightarrow \text{Binomial}$$

* $IL_p = 10 \log P_{LR}\big|_{\omega = \omega_p = \omega_c} = 2\ dB$

$$IL_p = 10 \log (1 + K^2) = 2\ dB$$

$$1 + K^2 = 10^{0.2}$$

$$\boxed{K = \sqrt{10^{0.2} - 1} = 0.764783}$$

* $IL_s = 10 \log P_{LR}\big|_{\omega = \omega_s} \geq 18\ dB$

$$10 \log \left[1 + K^2 \left(\frac{\omega_s}{\omega_c}\right)^{2N}\right] \geq 18 \text{ dB}$$

$$\left[1 + K^2 \left(\frac{\omega_s}{\omega_c}\right)^{2N}\right] \geq 10^{1.8}$$

$$K^2 \left(\frac{\omega_s}{\omega_c}\right)^{2N} \geq 10^{1.8} - 1$$

$$\left(\frac{\omega_s}{\omega_c}\right)^{2N} \geq \left[\frac{10^{1.8} - 1}{K^2}\right]$$

$$2N \ln\left(\frac{\omega_s}{\omega_c}\right) \geq \ln\left[\frac{10^{1.8} - 1}{K^2}\right]$$

$$N \geq \frac{\ln\left[(10^{1.8} - 1)/K^2\right]}{2 \ln(\omega_s/\omega_c)}$$

$$N \geq 4.84622$$

$$\boxed{N = 5} \longleftarrow \text{next integer}$$

لاحظ اذا ذكر في الـــ $IL = 18$ dB

من المقصود $IL \geq 18$ dB أن نحاول تحقيق القيمة

المطلوبة أو اكثر منها

لأن الـ (filter order) يجب أن يكون integer

$$\therefore IL_1 = 10 \log \left[1 + k^2 \left(\frac{\omega_1}{\omega_c} \right)^{2N} \right]$$

$$= 10 \log \left[1 + (0.764783)^2 \times \left(\frac{8.9}{5.5} \right)^{10} \right]$$

$$\boxed{IL_1 = 18.633399 \ dB}$$

∴ قمنا بتحقيق IL_1 اكبر من المطلوب

Example

- Determine N, K for a butter worth filter having $\omega_c = 5.5$ GHz, $\omega_s = 8.9$ GHz

$IL_s = 18$ dB

لاحظ في المسألة السابقة تم إعطاء معلومتين IL_p و IL_s

حتى نحصل على الـ K والـ N

أما في هذا المثال فإنه لدينا معلومة واحدة فقط $IL_s = 18$ dB

ففي هذه الحالة نفترض دائماً أن هناك معلومة أخرى $\boxed{IL_c = 3 \text{ dB}}$

$$\Rightarrow IL_c = 10 \log\left[1 + K^2 \left(\frac{\omega_c}{\omega_c}\right)^{2N}\right] = 3$$

$$= 10 \log\left[1 + K^2\right] = 3$$

$$\Rightarrow \boxed{K = 1}$$

$$IL_s = 10 \log\left[1 + \left(\frac{\omega_s}{\omega_c}\right)^{2N}\right] = 18$$

$$1 + \left(\frac{\omega_s}{\omega_c}\right)^{2N} = 10^{1.8}$$

$$2N \ln\left(\frac{\omega_s}{\omega_c}\right) = \ln\left[10^{1.8} - 1\right]$$

$$N = \frac{\ln[10^{1.8} - 1]}{2\ln(\omega_s/\omega_c)} = 4.289$$

$$\boxed{N = 5}$$

كل ما سبق كان ال $\boxed{\text{Analytical solution}}$

(II) Graphical solution

← يتم استخدام مجموعة من ال curves لإيجاد ال Filter order (N)

← كل نوع Filter له curves مختلفة

* Binomial "Butterworth" → has its own curves

* Chebyshev type 1 → has its own curves

وهكذا

← لاحظ كل نوع filter يتم رسمه اكثر من مره حسب قيمة ال K

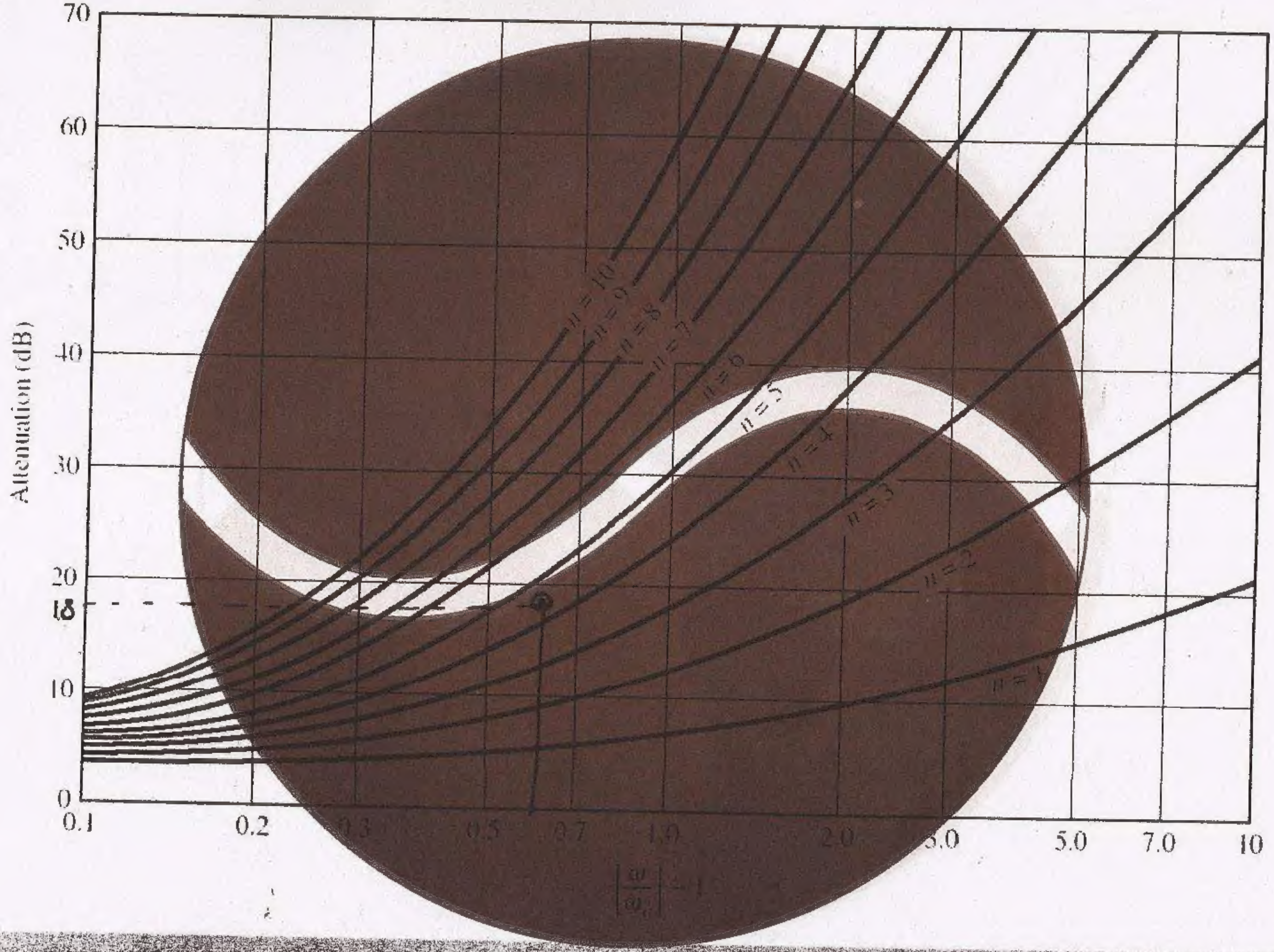
فمثلا ال curves في الصفحة التالي ← خاصة بحالة

(1) Butterworth (2) $K = 1$ $\begin{pmatrix} 3\ dB\ \text{at} \\ \omega = \omega_c \end{pmatrix}$

Low-Pass Filter Prototype

Design Curves for Maximally Flat LPF

12



Given the attenuation (insertion loss) at a frequency in the stopband frequency, the order of the filter N can be determined.

As indicated in the previous graph

$$\because \left|\frac{w_s}{w_c}\right| - 1 = \frac{8.9}{5.5} - 1 = \underline{\underline{0.618}}$$

$$\Rightarrow n > 4$$

$$\therefore \boxed{n = 5}$$

[2] Normalized low-Pass filter Prototype

* Given the filter order N

* and the filter type [Butterworth , chebychev . . .]

* the value of K

→ We need to get the value of the capacitors and inductors used in the prototype

* **There are two types of solutions**

[1] Analytical

[2] Using tables

Hint → to get the protype capacitors and inductors value we allways assume the following assumptions

[1] $R_s = 1$ [2] $w_c = 1$

Example

Get the prototype capacitors and inductor values for a Butterworth filter of 2^{nd} order

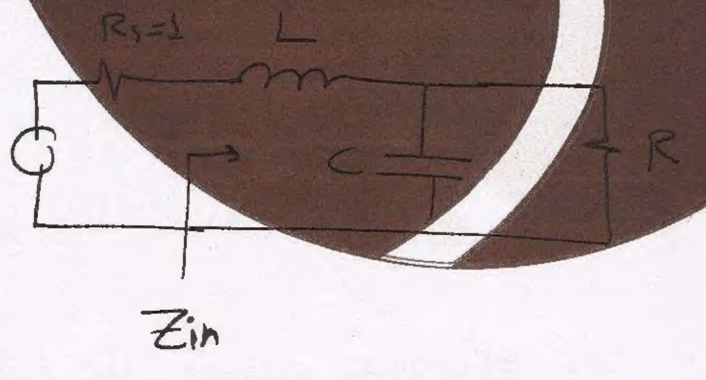
I Analytical solution

$\boxed{K=1}$ $\Leftarrow$ لاحظ لم يتم اعطاء مقدار K

→ for butterworth only

∴ 2^{nd} order filter $\Longrightarrow$ $N = 2$

∴ The prototype can be plotted as follow



$$\boxed{Z_{in} = jwL + \left(\frac{1}{jwc} // R\right)}$$